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# PLANETARY THEORY BASED UPON RIEMANN'S VECTORIALGEODESIC EQUATIONS OF MOTION AND GENERAL GRAVITATIONAL FIELD OF A STATIC HOMOGENEOUS SPHERICAL SUN

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## Abstract

We apply the Riemann's Vectorial Geodesic Equations of Motion to a planet in the general gravitational field of the Sun, regarding it as a static homogeneous spherical distribution of mass.

## 1:0 Introduction

In the book entitled ''**Riemannian Revolutions in Physics and Mathematics''** Prof. S.X.K Howusu published the *Riemann's Vectorial Geodesic Equations of Motion* and the *General Gravitational Field Equation* [1]. Then in the paper entitled ''*Generalized Dynamics Gravitation Field of a Static Homogenous Spherical Distribution* of mass'', we applied the *General Gravitational Field* Equation to derive the General Gravitation Field of the Static Homogeneous Spherical Distribution of mass [2].

In this paper we apply the Riemann's *Vectorial Geodesic Equations of Motion* to a planet in the *General Gravitational Field* of the Sun, regarding it as a static homogeneous spherical distribution of mass.

## 2:0 Analysis

According to the book entitled ‘‘**RIEMANNIAN REVOLUTIONS IN PHYSICS AND MATHEMATICS**’’ *Riemann's Vectorial Geodesic Equations of Motion* for particles of nonzero rest masses $m_0$ are given by [1]:

$$m_0 \underline{a}_R \;=\; \underline{F}_R^g + \underline{F}^{ng} \tag{1}$$

Or in component form:

$$m_0(\underline{a}_R)_{x^1} \;=\; (\underline{F}_R^g)_{x^1} + (\underline{F}^{ng})_{x^1} \tag{2}$$

$$m_0(\underline{a}_R)_{x^2} \;=\; (\underline{F}_R^g)_{x^2} + (\underline{F}^{ng})_{x^2} \tag{3}$$

$$m_0(\underline{a}_R)_{x^3} \;=\; (\underline{F}_R^g)_{x^3} + (\underline{F}^{ng})_{x^3} \tag{4}$$

$$m_0(\underline{a}_R)_{x^0} \;=\; (\underline{F}_R^g)_{x^0} + (\underline{F}^{ng})_{x^0} \tag{5}$$

Where $\underline{a}_R$ is the Riemann's linear acceleration vector defined in all orthogonal curvilinear co-ordinates $x^\mu$ by

$$(\underline{a}_R)_{x^3} = (g_{11})^{\frac{1}{2}}\left\{\ddot{x}^1 + \Gamma_{\mathrm{ij}}^1 \dot{x}^i \dot{x}^j + 2\Gamma_{\mathrm{i0}}^1 \dot{x}^i \dot{x}^0\right\} \tag{6}$$

$$(\underline{a}_R)_{x^2} = (g_{22})^{\frac{1}{2}}\left\{\ddot{x}^2 + \Gamma_{\mathrm{ij}}^2 \dot{x}^i \dot{x}^j + 2\Gamma_{\mathrm{i0}}^2 \dot{x}^i \dot{x}^0\right\} \tag{7}$$

$$(\underline{a}_R)_{x^3} = (g_{33})^{\frac{1}{2}}\left\{\ddot{x}^2 + \Gamma_{\mathrm{ij}}^3 \dot{x}^i \dot{x}^j + 2\Gamma_{\mathrm{i0}}^3 \dot{x}^i \dot{x}^0\right\} \tag{8}$$

$$(\underline{a}_R)_{x^0} = (g_{00})^{\frac{1}{2}}\left\{\ddot{x}^0 + \Gamma_{\mathrm{ij}}^0 \dot{x}^i \dot{x}^j + 2\Gamma_{\mathrm{i0}}^0 \dot{x}^i \dot{x}^0\right\} \tag{9}$$

And $\underline{F}_R^g$ is the Riemann's gravitation force vector defined by

$$\underline{F}_R^g = m_0 \underline{g}_R \tag{10}$$

Where $\underline{g}_R$ is the Riemann's gravitation intensity (or acceleration due to gravity) vector defined by

$$(\underline{g}_R)_{x^1} = -(g_{11})^{\frac{1}{2}} \Gamma_{00}^1 \dot{x}^0 \dot{x}^0 \tag{11}$$

$$(\underline{g}_R)_{x^2} = -(g_{22})^{\frac{1}{2}}\Gamma_{00}^{2}\dot{x}^0\dot{x}^0 \tag{12}$$

$$(\underline{g}_R)_{x^3} = -(g_{33})^{\frac{1}{2}}\Gamma_{00}^{3}\dot{x}^0\dot{x}^0 \tag{13}$$

$$(\underline{g}_R)_{x^0} = -(g_{00})^{\frac{1}{2}}\Gamma_{00}^{0}\dot{x}^0\dot{x}^0 \tag{14}$$

and $\underline{u}$ is the instantaneous linear velocity vector in space-time given by

$$(\underline{u})_{x^0} = (g_{00})^{\frac{1}{2}}\dot{x}^0 \tag{15}$$

$$(\underline{u})_{x^1} = (g_{11})^{\frac{1}{2}}\dot{x}^1 \tag{16}$$

$$(\underline{u})_{x^2} = (g_{22})^{\frac{1}{2}}\dot{x}^2 \tag{17}$$

$$(\underline{u})_{x^3} = (g_{33})^{\frac{1}{2}}\dot{x}^3 \tag{18}$$

And $\mu_R$ is the Riemannian linear space speed given by

$$u_R^2 = g_{ij}\dot{x}^i\dot{x}^j \tag{19}$$

Where $x^\mu$ is the space-time position tensor, $g_{uv}$ is the one and only unique metric tensor for all gravitational fields in nature, $\Gamma_{uv}^{\alpha}$ is the Christoffel symbol of the second kind (or coefficient of affine connection) pseudo tensor and a dot denotes one differentiation with respect to proper time, and $\underline{F}^{ng}$ is the non-gravitational force acting on the particle.

If we consider the Sun as a static homogeneous spherical distribution of mass then one appropriate co-ordinate system is the spherical polar. And in the Einstein spherical polar co-ordinates the one and only one (unique) metric tensor for all gravitational fields in nature is given by [1]:

$$x^0 = ct \tag{20}$$

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$$g_{00} = -\left(1 + \tfrac{2}{c^2} f\right) \tag{21}$$

$$g_{11} = \left(1 + \tfrac{2}{c^2} f\right)^{-1} \tag{22}$$

$$g_{22} = r^2 \tag{23}$$

$$g_{33} = r^2 sin^2\theta \tag{24}$$

$$g_{\mu v} = 0 \text{ otherwise} \tag{25}$$

Hence the complete Riemann's linear acceleration vector is given explicitly by

$$\left(\underline{a}_R\right)_{x^0} = c\left(1 + \frac{2}{c^2} f\right)\ddot{t} + \frac{2}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} f_{,r}\dot{t}\dot{r} + \frac{2}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} f_{,\theta}\dot{t}\dot{\theta}$$
$$+ \tfrac{2}{c^2}\left(1 + \tfrac{2}{c^2} f\right)^{-\frac{1}{2}} f_{,\phi}\dot{t}\dot{\phi} + \tfrac{2}{c^2}\left(1 + \tfrac{2}{c^2} f\right)^{-\frac{1}{2}} f_{,r}\dot{t}\dot{r}^2 \tag{26}$$

and



$$\left(\underline{a}_R\right)_r = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}}\ddot{r} - \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} r\dot{\theta}^2 - \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} rsin\theta\dot{\phi}^2$$
$$- \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-\frac{3}{2}} f_{,r}\,\dot{r}^2 - \frac{2}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} f_{,t}\dot{t}\dot{r}$$
$$- \tfrac{2}{c^2}\left(1 + \tfrac{2}{c^2} f\right)^{-\frac{3}{2}} f_{,\phi}\dot{r}\dot{\phi} \tag{27}$$

and

$$\left(\underline{a}_R\right)_r = r\ddot{\theta} + 2\dot{r}\dot{\theta} - rsin\theta cos\theta\dot{\phi}^2 + \tfrac{1}{c^2}\left(1 + \tfrac{2}{c^2} f\right)^{-\frac{3}{2}} f_{,\theta}\,\dot{r}^2 \tag{28}$$

And

$$\left(\underline{a}_R\right)_r = rsin\theta\ddot{\phi} + 2sin\theta\dot{r}\phi + 2rcos\theta\dot{\theta}\dot{\phi} + \frac{1}{c^2 rsin\theta} f_{,\phi}\,\dot{r}^2 \tag{29}$$

where $f$ is the gravitational scalar potential.

Similarly the complete Riemann's gravitational intensity (or acceleration due to gravity) vector is given explicitly by

$$(\underline{g}_R)_{x^0} = -\frac{1}{c}\dot{t}^2\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\frac{\partial f}{\partial t} \tag{30}$$

$$(\underline{g}_R)_r = -\frac{1}{c}\dot{t}^2\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\frac{\partial f}{\partial r} \tag{31}$$

$$(\underline{g}_R)_\theta = -\dot{t}^2\frac{1}{r}\frac{\partial f}{\partial \theta} \tag{32}$$

$$(\underline{g}_R)_\phi = -\dot{t}^2\frac{1}{rsin}\frac{\partial f}{\theta\partial \phi} \tag{33}$$

Similarly the complete linear velocity vector is given explicitly by

$$(\underline{u})_{x^0} = c\left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}}\dot{t} \tag{34}$$

$$(\underline{u})_r = \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{r} \tag{35}$$

$$(\underline{u})_\theta = r\dot{\theta} \tag{36}$$

$$(\underline{u})_\phi = rsin\theta\dot{\phi} \tag{37}$$

Similarly the complete Riemann's space speed is given explicitly by

$$u_R^2 = \left(1 + \frac{2}{c^2}f\right)^{-1}\dot{r}^2 + \dot{r}^2\dot{\theta}^2 + r^2sin^2\theta\dot{\phi}^2 \tag{38}$$

Finally in the case of a static homogenous spherical Sun of radius R and total rest mass $M_0$ the general gravitational scalar potential is given by

$$f^+(r,\theta,\phi,x^0) = \frac{A}{r} - \frac{A^2}{c^2r^2} + \frac{A^3}{c^4r^3} - \frac{3A^4}{c^2r^2} + \ldots ; \quad r > R \tag{39}$$

$$f^-(r,\theta,\phi,x^0) = B + \frac{GM_0}{2R}\left(\frac{r}{R}\right)^2 - \frac{G^2M_0^2}{4c^2R^2}\left(\frac{r}{R}\right)^4 + \frac{G^3M_0^3}{4c^6R^3}\left(\frac{r}{R}\right)^6 \ldots ; \quad r < R \tag{40}$$

here

$$A \approx -GM_0\left(1+\frac{GM_0}{c^2R}+\frac{3G^2M_0^2}{2c^4R^2}+\ ...\right) \quad (41)$$

And

$$B \approx -\frac{3GM_0}{2R}\left(1+\frac{GM_0}{2c^2R}+\frac{5G^2M_0^2}{6c^4R^2}+\ ...\right) \quad (41)$$

where G is the universal gravitational constant [2]. It therefore follows that for a planet moving in the gravitation field exterior to the static homogenous spherical Sun only,

$$\underline{F}^{ng} \equiv \underline{0} \quad (43)$$

And the equations of motion are given by

$$\underline{a}_R = -\underline{g}_R \quad (44)$$

Where the gravitation scalar potential f is given by (39) and the corresponding Riemann's linear acceleration vector is given by

$$(\underline{a}_R)_{x^0} = c\left(1+\frac{2}{c^2}f\right)\ddot{t}+\frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}f_{,r}\dot{t}\dot{r} \quad (45)$$

$$(\underline{a}_R)_r = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\ddot{r}-\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}}r\dot{\theta}^2-\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}}r sin^2\theta\dot{\phi}^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}}f_{,r}\,\dot{r}^2 \quad (46)$$

$$(\underline{a}_R)_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r sin\theta cos\theta\dot{\phi}^2 \quad (47)$$

$$(\underline{a}_R)_\phi = r sin\theta\ddot{\phi} + 2sin\theta\dot{r}\dot{\phi} + 2r cos\theta\dot{\theta}\,\dot{\phi} \quad (48)$$

$$(\underline{g}_R)_{x^0} = 0 \quad (49)$$

$$(\underline{g}_R)_r = -\dot{t}^2\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} f_{,r} \qquad (50)$$

$$(\underline{g}_R)_\theta = 0 \qquad (51)$$

$$(\underline{g}_R)_\phi = 0 \qquad (52)$$

And $\underline{u}$ is given by (35) – (37) and $u_R^2$ is given by (38).

Now towards the solution of the planetary equations it may be noted that the time equation integrates exactly to yield the result:

$$\dot{t} = A\left(1+\frac{2}{c^2}f\right)^{-1} \qquad (53)$$

where A is an arbitrary constant.

Therefore from the well known condition that

$$\dot{t} \rightarrow 1 as f \rightarrow 0 \qquad (54)$$

It follows that

$$A = 1 \qquad (55)$$

Consequently $\dot{t}$ given by (53) can be substituted into all the other equations of motion before analyzing them further.

## 3:0 Summary and Conclusion

In this paper we formulated the planetary equation based upon Riemann's Vectorial Geodesic Equations of Motion [1] and the assumption of a static homogeneous spherical Sun. Then we solved the time equation to obtain the time dilation formula of the planets. Therefore the space equations are ready for analysis and subsequent description of the motions of the planets.

It may be noted that precisely as in the case of the pure Newtonian planetary equations the analysis is most simple if the planet is modeled as confined to the equatorial plane of the Sun.

It is most interesting and instructive to note that the planetary equations based upon Riemann's Vectorial Geodesic Equations of Motion in the gravitational field of a static homogeneous spherical Sun contain post Newton or pure Riemannian corrections of all orders of $c^{-2}$ which are henceforth exposed for experimental detection and applications.

It is also interesting and instructive to note that by comparison within the earlier papers entitled "On the Gravitational of Moving Bodies'' [3] and ‘‘Golden Dynamical Equations of Motion'' [4]. It follows that the planetary equations according to Riemann's Vectorial Geodesic Equations of Motion in this paper will result in the phenomenon of anomalous orbital precession of the planets in the Solar System as excellently as any previous theory, a fortiori.

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